1 Exchange Rate

1.1 Nominal Exchange Rate

The nominal exchange rate is the rate at which a person can trade the currency of one country for the currency of another (How much foreign currency/domestic currency).

1.2 Real Exchange Rate

The real exchange rate is the rate at which a person can trade the goods and services of one country for the goods and services of another (How much foreign G&S/domestic G&S).
The relationship between real and nominal exchange rates is given by the following formula:

$$\text{Real Exchange Rate} = \frac{\text{Nominal Exchange Rate} * \text{Domestic Price} * 1}{\text{Foreign Price}}$$

Intuition:

- The real exchange rate represents how many units of foreign goods and services you can obtain by giving up one unit of domestic goods and services.
- By giving up **1** unit of domestic G&S at the "**Domestic Price**", you receive: **Nominal Exchange Rate * Domestic Price * 1** unit of foreign currency. These foreign currencies can be exchanged for $\frac{\text{Nominal Exchange Rate} * \text{Domestic Price} * 1}{\text{Foreign Price}}$ units of foreign G&S.

1.3 Purchasing Power Parity

Purchasing-power parity is a theory of exchange rates stating that a unit of any given currency should be able to buy the **same quantity of goods** in all countries; i.e., a unit of currency must have the same **real value** in every country.
Intuition: PPP implied real exchange rate equals to **1**.

2 Solow Growth Model

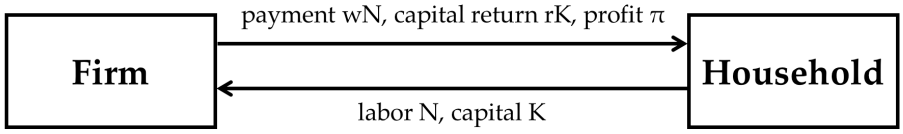


Figure 1: Solow Model

2.1 Firm

At each period t, Firms rent capital at a rate r_t and pay workers a wage of w_t(Those variables are exogenous to the firm). It chooses K_t and N_t to maximize the profit.

Production function: $Y_t = A_t F(K_t, N_t)$

- Production function is CRS, or homogenous of degree one (To get rid of the size problem)

- Review: A function $F(K, N)$ is **homogeneous of degree** θ if for all $\lambda > 0$:

$$F(\lambda K, \lambda N) = \lambda^{\theta} F(K, N)$$

- Important properties:
  - $F_K > 0$, $F_N > 0$
  - $F_{KK} < 0$, $F_{NN} < 0$
  - $F_{KN} > 0$
  - $F(K, 0) = 0$, $F(0, N) = 0$

Firm's Problem: Firm chooses K_t and N_t to maximize:

$$\Pi_t = A_t F(K_t, N_t) - r_t K_t - w_t N_t$$

- FOCs:

$$[N_t]: \quad w_t = A_t F_N(K_t, N_t)$$

,

$$[K_t]: \quad r_t = A_t F_K(K_t, N_t)$$

Intuition: Marginal cost of capital/labor = marginal output of capital/labor

- Zero-profit:
  The profit function is:

$$\Pi_t = A_t F(K_t, N_t) - r_t K_t - w_t N_t \tag{1}$$

By Euler's theorem:

$$\rho F(K_t, N_t) = F_K(K_t, N_t) K_t + F_N(K_t, N_t) N_t$$

We assumed $\rho = 1$(Production function is homogenous of degree 1):

$$A_t F(K_t, N_t) = A_t F_K(K_t, N_t) K_t + A_t F_N(K_t, N_t) N_t$$

By FOCs, $r_t = A_t F_K(K_t, N_t)$, $w_t = A_t F_N(K_t, N_t)$:

$$A_t F(K_t, N_t) = r_t K_t + w_t N_t \tag{2}$$

Combine (1) and (2):

$$\Pi_t = r_t K_t + w_t N_t - r_t K_t - w_t N_t$$

$$\Pi_t = 0$$

2.2 Household

At each period t, households rent K_t units of capital stock to firms at the rate r_t and supply N_t units of labor to firms at the wage w_t. It save a constant fraction s of the income as investment I_t.

Household Budget Constraint:

$$\underbrace{C_t + I_t}_{\text{Expenditure}} \leq \underbrace{\overbrace{\underbrace{w_t N_t}_{\text{Labor income}} + \underbrace{r_t K_t}_{\text{Capital income}}}^{Y_t} + \underbrace{\Pi_t}_{\text{Profits}}}_{\text{Income}}$$

- Budget constraint is binding when maximizing the utility: $C_t + I_t = w_t N_t + R_t K_t + \Pi_t$
- Given $\Pi_t = 0$, $Y_t = r_t K_t + w_t N_t$,
$$C_t + I_t = Y_t$$
, this is a closed economy(no exports and imports) with no government spending(no G) or taxes(no τ)
- Assume a constant fraction of income is saved,
$$I_t = sY_t$$
$$C_t = (1-s)Y_t$$

Law of Motion of Capital: $K_{t+1} = I_t + (1-\delta)K_t$

$$\underbrace{K_{t+1}}_{\text{Capital for the Next Period's Production}} = \underbrace{\underbrace{I_t}_{\text{Investment, } I_t = sY_t} + \underbrace{(1-\delta)K_t}_{\text{Depreciated capital}}}_{\text{Two sources from this period: investment and remaining capital}}$$

2.3 Key Equations of Solow Model

- Production function: $Y_t = A_t F(K_t, N_t)$
- Resource constraint: $Y_t = C_t + I_t$
- Law of motion: $K_{t+1} = I_t + (1-\delta)K_t$
- Savings: $I_t = sY_t$
- FOCs: $w_t = AF_N(K_t, N_t)$, $r_t = AF_K(K_t, N_t)$

3 Exercise: Simple two-country model[1]

Suppose there is only one period and there are two countries A and B. In country A, there is one representative household who owns capital stock $K_A = \bar{K} > 0$ and one unit of labor $N_A = 1$. There is one representative firm who produces by renting the capital with rental rate r_A and by hiring labor with wage w_A in country A. The production function is

$$Y_A = A_A(\hat{K}_A)^\alpha(\hat{N}_A)^{1-\alpha}, \quad \alpha \in (0,1)$$

Country B is symmetric except

$$A_A < A_B.$$

1. Write down the firm's profit in country A and country B. Derive the FOCs with respect to the capital choice $\hat{K}_A$ and $\hat{K}_B$.

[1]Based on Handout from Andy Lehrer, modified by Xuexin

2. Suppose there is no international capital flow, i.e., the firm in country A only rents the capital from country A household and the firm in country B only rents the capital from country B household. Calculate the market rental rate of capital r_A^* and r_B^* under no international capital flow.

3. Suppose there is an international capital renting market with rental rate r^* with $r_A^* < r^* < r_B^*$. Then which country rents from the rest of the world? Which country rents to the rest of the world?

4. Gourinchas and Jeanne (2013) show that cross-country correlation between capital inflow and productivity growth is negative among developing countries. That is, countries with higher productivity and investment opportunity experience capital outflow. Is this finding consistent with the model prediction?

4 Solution: Simple two-country model

Suppose there is only one period and there are two countries A and B. In country A, there is one representative household who owns capital stock $K_A = \bar{K} > 0$ and one unit of labor $N_A = 1$. There is one representative firm who produces by renting the capital with rental rate r_A and by hiring labor with wage w_A in country A. The production function is

$$Y_A = A_A(\hat{K}_A)^{\alpha}(\hat{N}_A)^{1-\alpha}, \quad \alpha \in (0,1)$$

Country B is symmetric except

$$A_A < A_B.$$

1. Write down the firm's profit in country A and country B. Derive the FOCs with respect to the capital choice $\hat{K}_A$ and $\hat{K}_B$.

   The profits are

   $$\Pi_A = A_A(\hat{K}_A)^{\alpha}(\hat{N}_A)^{1-\alpha} - r_A\hat{K}_A - w_A\hat{N}_A$$

   $$\Pi_B = A_B(\hat{K}_B)^{\alpha}(\hat{N}_B)^{1-\alpha} - r_B\hat{K}_B - w_B\hat{N}_B$$

   Then we can derive the FOCs as follows

   $$\alpha A_A(\hat{K}_A)^{\alpha-1}(\hat{N}_A)^{1-\alpha} = r_A$$

   $$\alpha A_B(\hat{K}_B)^{\alpha-1}(\hat{N}_B)^{1-\alpha} = r_B$$

2. Suppose there is no international capital flow, i.e., the firm in country A only rents the capital from country A household and the firm in country B only rents the capital from country B household. Calculate the market rental rate of capital r_A^* and r_B^* under no international capital flow.

   Since there is no international capital flow and there is one unit of labor supply in each country, capital and labor markets clear with

   $$\hat{K}_A = \bar{K}, \quad \hat{K}_B = \bar{K}, \quad \hat{N}_A = 1, \quad \hat{N}_B = 1$$

   Substituting those into the firms' FOC gives

   $$r_A^* = \alpha A_A(\bar{K})^{\alpha-1}, \quad r_B^* = \alpha A_B(\bar{K})^{\alpha-1}$$

3. Suppose there is an international capital renting market with rental rate r^* with $r_A^* < r^* < r_B^*$. Then which country rents from the rest of the world? Which country rents to the rest of the world?
Since the world rental rate is given by r^* and labor supply is still 1 in each country, a firm in each country chooses to rent the capital stock satisfying

$$\alpha A_A(\hat{K}_A)^{\alpha-1} = r^*$$
$$\alpha A_B(\hat{K}_B)^{\alpha-1} = r^*$$

For country A, since

$$\alpha A_A(\bar{K})^{\alpha-1} = r_A^* < r^* = \alpha A_A(\hat{K}_A)^{\alpha-1}$$

The capital stock held by household $\bar{K}$ is larger than the capital stock rented by firm $\hat{K}_A$. Thus, country A will rent to the rest of the world.

For country B, since

$$\alpha A_B(\bar{K})^{\alpha-1} = r_B^* > r^* = \alpha A_B(\hat{K}_B)^{\alpha-1}$$

The capital stock held by household $\bar{K}$ is smaller than the capital stock rented by firm $\hat{K}_B$. Thus, country B will rent from the rest of the world.

4. Gourinchas and Jeanne (2013) show that cross-country correlation between capital inflow and productivity growth is negative among developing countries. That is, countries with higher productivity and investment opportunity experience capital outflow. Is this finding consistent with the model prediction?
In the model, more productive country (Country B) rents from the rest of the world (Capital inflow) and less productive country (Country A) rents to the rest of the world (Capital outflow). Therefore, the model prediction is at odds with the empirical pattern.